1.)

(a) $\boldsymbol{WP} = \begin{pmatrix} 13 \\ 5 \\ 6 \end{pmatrix}$ A1A1A1 N3

Note: *Award A1 for each correct element.*

(b) ***Note:*** *The first two steps may be done in any order.*

subtracting (A1)

e.g. $\begin{pmatrix} 26 \\ 12 \\ 10 \end{pmatrix} - 2\boldsymbol{WP}$

multiplying $\boldsymbol{WP}$ by 2 (A1)

e.g. $\begin{pmatrix} 26 \\ 10 \\ 12 \end{pmatrix}$

$\boldsymbol{S} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$ A1 N2

[6]

2.) (a) evidence of multiplying (M1)

e.g. one correct element

$\boldsymbol{AB} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ A1A1 N3

(b) **METHOD 1**

evidence of multiplying by $\boldsymbol{A}$ (on left or right) (M1)

e.g. $\boldsymbol{AA}^{-1}\boldsymbol{X} = \boldsymbol{AB}$, $\boldsymbol{X} = \boldsymbol{AB}$

$\boldsymbol{X} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ (accept $x = -15$, $y = 5$) A1 N2

METHOD 2

attempt to set up a system of equations (M1)

e.g. $\frac{4x+2y}{10} = -5, \frac{-3x+y}{10} = 5$

$\boldsymbol{X} = \begin{pmatrix} -15 \\ 5 \end{pmatrix}$ (accept $x = -15$, $y = 5$) A1 N2

[5]

3.) (a) (i) $\boldsymbol{AB} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ (= $4\boldsymbol{I}$) A2 N2

(ii) $\boldsymbol{A}^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}, \frac{1}{4}\boldsymbol{B}, \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{5}{4} \end{pmatrix}$ A1 N1

(b) **METHOD 1**

$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{A}^{-1}\boldsymbol{C}$ (M1)

$= \frac{1}{4}\begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}\begin{pmatrix} 8 \\ -4 \end{pmatrix} \quad \left(\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{5}{4} \end{pmatrix}\begin{pmatrix} 8 \\ -4 \end{pmatrix}\right)$ A1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$ A1A1 N3

METHOD 2

$5x + y = 8$, $6x + 2y = -4$ A1
for work towards solving **their** system (M1)

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$ A1A1 N3

[7]

4.) (a) **METHOD 1**

$\boldsymbol{M} = (\boldsymbol{M}^{-1})^{-1}$ (M1)

$\boldsymbol{M} = \frac{1}{10}\begin{pmatrix} 2 & 0 \\ -1 & 5 \end{pmatrix}$ A1A1 N3

METHOD 2

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (M1)

$5a + b = 1$, $2b = 0$, $5c + d = 0$, $2d = 1$ (A1)

$\boldsymbol{M} = \begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}$ A1 N3

(b) **METHOD 1**

evidence of appropriate approach (M1)
e.g. $\boldsymbol{X} = \boldsymbol{M}^{-1}\boldsymbol{B}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \end{pmatrix}$ A1

$= \begin{pmatrix} 5 \\ 15 \end{pmatrix}$ A1 N2

METHOD 2

evidence of appropriate approach (M1)

e.g. $\begin{pmatrix} 0.2 & 0 \\ -0.1 & 0.5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$0.2x = 1$, $-0.1x + 0.5y = 7$ A1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \end{pmatrix}$ A1 N2

[6]

5.) (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 3 & 2 & -3 \\ 2 & 1 & -2 \\ -8 & -6 & 9 \end{pmatrix}$ A2 N2

(b) evidence of subtracting matrices (M1)

e.g. $\begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix}, \begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix}, \boldsymbol{D} - \boldsymbol{C}$

evidence of multiplying **on left** by $\boldsymbol{A}^{-1}$ (M1)

e.g. $\boldsymbol{A}^{-1}\boldsymbol{AB}, \boldsymbol{A}^{-1}(\boldsymbol{D} - \boldsymbol{C}), \begin{pmatrix} 3 & 2 & -3 \\ 2 & 1 & -2 \\ -8 & -6 & 9 \end{pmatrix}\begin{pmatrix} 10 & 4 & -8 \\ 1 & 2 & -12 \\ 10 & 5 & -15 \end{pmatrix}$

$\boldsymbol{B} = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 0 & 2 \\ 4 & 1 & 1 \end{pmatrix}$ A2 N3

[6]

6.) (a) evidence of addition (M1)

e.g. at least two correct elements

$\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}$ A1 N2

(b) evidence of multiplication (M1)

e.g. at least two correct elements

$-3\boldsymbol{A} = \begin{pmatrix} -3 & -6 \\ -9 & 3 \end{pmatrix}$ A1 N2

(c) evidence of matrix multiplication (in correct order) (M1)

e.g. $\boldsymbol{AB} = \begin{pmatrix} 1(3)+2(-2) & 1(0)+2(1) \\ 3(3)+(-1)(-2) & 3(0)+(-1)(1) \end{pmatrix}$

$\boldsymbol{AB} = \begin{pmatrix} -1 & 2 \\ 11 & -1 \end{pmatrix}$ A2 N3

[7]

7.) (a) evidence of correct method (M1)

e.g. at least 1 correct element (must be in a 2 × 2 matrix)

$\boldsymbol{AB} = \begin{pmatrix} -2-2q & 0 \\ -6+pq & 3+\frac{p}{2} \end{pmatrix}$ A1 N2

(b) **METHOD 1**

evidence of using $\boldsymbol{AB} = \boldsymbol{I}$ (M1)

2 correct equations A1A1

e.g. $-2-2q = 1$ and $3 + \frac{p}{2} = 1$, $-6 + pq = 0$

$p = -4$, $q = -\frac{3}{2}$ A1A1 N1N1

METHOD 2

finding $\boldsymbol{A}^{-1} = \frac{1}{p+6}\begin{pmatrix} p & 2 \\ -3 & 1 \end{pmatrix}$ A1

evidence of using $\boldsymbol{A}^{-1} = \boldsymbol{B}$ (M2)

e.g. $\frac{2}{p+6} = 1$ and $-\frac{3}{p+6} = q$, $\frac{p}{p+6} = -2$ and $-\frac{3}{p+6} = q$

$p = -4$, $q = -\frac{3}{2}$ A1A1 N1N1

[7]

8.) Let $\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(2)

The matrix $\boldsymbol{B}$ satisfies the equation $\left(\boldsymbol{I} - \frac{1}{2}\boldsymbol{B}\right)^{-1} = \boldsymbol{A}$, where $\boldsymbol{I}$ is the 3 × 3 identity matrix.

(b) (i) Show that $\boldsymbol{B} = -2(\boldsymbol{A}^{-1} - \boldsymbol{I})$.

(ii) Find $\boldsymbol{B}$.

(iii) Write down det $\boldsymbol{B}$.

(iv) **Hence**, explain why $\boldsymbol{B}^{-1}$ exists.

(6)

Let $\boldsymbol{BX} = \boldsymbol{C}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find $\boldsymbol{X}$.

(ii) Write down a system of equations whose solution is represented by $\boldsymbol{X}$.

(5)
(Total 13 marks)

9.) (a) Attempt to multiply *e.g.* $\begin{pmatrix} 1+0 & -2-6 \\ 0+0 & 0+9 \end{pmatrix}$ (M1)

$\boldsymbol{A}^2 = \begin{pmatrix} 1 & -8 \\ 0 & 9 \end{pmatrix}$ A1 N2

(b) $3\boldsymbol{X} + \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$ (M1)

$3\boldsymbol{X} = \begin{pmatrix} -4 & 6 \\ 2 & -2 \end{pmatrix}$ (A1)

$\boldsymbol{X} = \frac{1}{3}\begin{pmatrix} -4 & 6 \\ 2 & -2 \end{pmatrix}$ A1 N2

[5]

10.) $\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (A1)

$\boldsymbol{M}^2 = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix}$ A2

$6\boldsymbol{M} = \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix}$ A1

$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ A1

$k = 5$ A1 N2

[6]

11.) (a) Attempting to multiply matrices(M1)

$$\begin{pmatrix}1 & x & -1\\ 3 & 1 & 4\end{pmatrix}\begin{pmatrix}3\\ x\\ 2\end{pmatrix}=\begin{pmatrix}3+x^2-2\\ 9+x+8\end{pmatrix}\left(=\begin{pmatrix}1+x^2\\ 17+x\end{pmatrix}\right)$$ A1A1 N3

(b) Setting up equation M1

eg $2\begin{pmatrix}1+x^2\\ 17+x\end{pmatrix}=\begin{pmatrix}20\\ 28\end{pmatrix}$, $\begin{pmatrix}2+2x^2\\ 34+2x\end{pmatrix}=\begin{pmatrix}20\\ 28\end{pmatrix}$, $\begin{pmatrix}1+x^2\\ 17+x\end{pmatrix}=\begin{pmatrix}10\\ 14\end{pmatrix}$

$\begin{matrix}2+2x^2=20\\ 34+2x=28\end{matrix}$ $\begin{pmatrix}1+x^2=10\\ 17+x=14\end{pmatrix}$ (A1)

$x=-3$ A1 N2

[6]

12.) (a) (i) $\boldsymbol{A}^{-1}=\begin{pmatrix}0 & \frac{1}{2}\\ \frac{1}{2} & 0\end{pmatrix}$ A2 N2

(ii) $\boldsymbol{A}^2=\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}$ A2 N2

(b) $\begin{pmatrix}0 & 4\\ 4 & 0\end{pmatrix}+\begin{pmatrix}p & 2\\ 0 & q\end{pmatrix}=\begin{pmatrix}2 & 6\\ 4 & 3\end{pmatrix}$ (M1)

$p=2$, $q=3$ A1A1 N3

(c) Evidence of attempt to multiply (M1)

eg $\boldsymbol{A}^{-1}\boldsymbol{B}=\begin{pmatrix}0 & \frac{1}{2}\\ \frac{1}{2} & 0\end{pmatrix}\begin{pmatrix}2 & 2\\ 0 & 3\end{pmatrix}$

$\boldsymbol{A}^{-1}\boldsymbol{B}=\begin{pmatrix}0 & \frac{3}{2}\\ 1 & 1\end{pmatrix}$ $\left(\text{accept}\begin{pmatrix}0 & \frac{1}{2}q\\ \frac{1}{2}p & 1\end{pmatrix}\right)$ A1 N2

(d) Evidence of correct approach (M1)

eg $\boldsymbol{X}=\boldsymbol{A}^{-1}\boldsymbol{B}$, setting up a system of equations

$\boldsymbol{X}=\begin{pmatrix}0 & \frac{3}{2}\\ 1 & 1\end{pmatrix}$ $\left(\text{accept}\begin{pmatrix}0 & \frac{1}{2}q\\ \frac{1}{2}p & 1\end{pmatrix}\right)$ A1 N2

[11]

13.) (a) (i) $a=5$ A1 N1

(ii) $b+9=4$ (M1)

$b=-5$ A1 N2

(b) Comparing elements $3(2) - 5(q) = -9$ M1

$q = 3$ A2 N2

[6]

14.) (a) (i) $S_4 = 20$ A1 N1

(ii) $u_1 = 2$, $d = 2$ (A1)

Attempting to use formula for S_n M1

$S_{100} = 10100$ A1 N2

(b) (i) $\boldsymbol{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ A2N2

(ii) For writing $\boldsymbol{M}^3$ as $\boldsymbol{M}^2 \times \boldsymbol{M}$ or $\boldsymbol{M} \times \boldsymbol{M}^2$ $\left(\text{or} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\right)$ M1

$\boldsymbol{M}^3 = \begin{pmatrix} 1+0 & 4+2 \\ 0+0 & 0+1 \end{pmatrix}$ A2

$\boldsymbol{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$ AG N0

(c) (i) $\boldsymbol{M}^4 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ A1N1

(ii) $\boldsymbol{T}^4 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 4 & 20 \\ 0 & 4 \end{pmatrix}$ A1A1 N3

(d) $\boldsymbol{T}_{100} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \ldots + \begin{pmatrix} 1 & 200 \\ 0 & 1 \end{pmatrix}$ (M1)

$= \begin{pmatrix} 100 & 10100 \\ 0 & 100 \end{pmatrix}$ A1A1 N3

[16]

15.) (a) $2\boldsymbol{A} = \begin{pmatrix} 6 & 4 \\ 2k & 8 \end{pmatrix}$ (A1)

$2\boldsymbol{A} - \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 2k-1 & 5 \end{pmatrix}$ A2 N3

(b) Evidence of using the definition of determinant (M1)

Correct substitution (A1)

eg $4(5) - 2(2k - 1)$, $20 - 2(2k - 1)$, $20 - 4k + 2$

$\det(2\boldsymbol{A} - \boldsymbol{B}) = 22 - 4k$ A1 N3

[6]

16.) (a) $\boldsymbol{A}+\boldsymbol{B}=\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}+\begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$

$=\begin{pmatrix} a+1 & b \\ c+d & e \end{pmatrix}$ A2 2

(b) $\boldsymbol{AB}=\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}+\begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$ A1A1A1A1 4

***Note**: Award N2 for finding* $\boldsymbol{BA}=\begin{pmatrix} a & b \\ ad+ce & bd \end{pmatrix}$.

[6]

17.) (a) $3\boldsymbol{Q}=\begin{pmatrix} -4 & 8 \\ 2 & 14 \end{pmatrix}-\begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$ (A1)

$3\boldsymbol{Q}=\begin{pmatrix} -9 & 6 \\ 3 & 14-a \end{pmatrix}$ (A1)

$\boldsymbol{Q}=\begin{pmatrix} -3 & 2 \\ 1 & \frac{14-a}{3} \end{pmatrix}$ (A1) (N3) 3

(b) $\boldsymbol{CD}=\begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$

$=\begin{pmatrix} -14 & -4+4a \\ -2 & 2+7a \end{pmatrix}$ (A1)(A1)(A1)(A1) (N4) 4

(c) $\det \boldsymbol{D}=5a+2$ (may be implied) (A1)

$\boldsymbol{D}^{-1}=\frac{1}{5a+2}\begin{pmatrix} a & -2 \\ 1 & 5 \end{pmatrix}$ (A1) (N2) 2

[9]

18.) (a) $\boldsymbol{A}^{-1}=\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{7}{3} & \frac{5}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -7 & 5 \end{pmatrix}$ or $\begin{pmatrix} 0.667 & -0.333 \\ -2.33 & -1.67 \end{pmatrix}$ (A1)(A1) (N2)

(b) $\boldsymbol{AX}=\boldsymbol{C}-\boldsymbol{B}$ (may be implied) (A1)

$\boldsymbol{X}=\boldsymbol{A}^{-1}(\boldsymbol{C}-\boldsymbol{B})$ (A1)

$\boldsymbol{D}=\boldsymbol{C}-\boldsymbol{B}$

$=\begin{pmatrix} 7 & -11 \\ 11 & -13 \end{pmatrix}$ (A1) (N3)

(c) $\boldsymbol{X}=\begin{pmatrix} 1 & -3 \\ 2 & 4 \end{pmatrix}$ (A2) (N2)

[7]

19.) $\begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}\boldsymbol{X} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$

$\begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ (M1)

Pre-multiply by inverse of $\begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}$ (M1)

$\boldsymbol{X} = \frac{1}{33}\begin{pmatrix} 7 & -1 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ (A1)(A1)

Note: *Award (A1) for determinant, (A1) for matrix* $\begin{pmatrix} 7 & -1 \\ 5 & 4 \end{pmatrix}$.

$= \frac{1}{33}\begin{pmatrix} 28 & 59 \\ 20 & 28 \end{pmatrix}$ (A1)(A1)(A1)(A1)

$\left(\Rightarrow a = \frac{28}{33}, b = \frac{59}{33}, c = \frac{20}{33}, d = \frac{28}{33}\right)$

OR

$\begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ (A1)

$\begin{pmatrix} 3a + c & 3b + d \\ -5a + 6c & -5b + 6d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ (A1)

$4a + c = 4$
$-5a + 7c = 0$ (A1)
$4b + d = 8$
$-5b + 7d = -3$ (A1)

Notes: *Award (A1) for each pair of equations.*
*Allow **ft** from their equations.*

$a = \frac{28}{33}, b = \frac{59}{33}, c = \frac{20}{33}, d = \frac{28}{33}$ (A1)(A1)(A1)(A1)

Note: *Award (A0)(A0)(A1)(A1) if the final answers are given as decimals ie 0.848, 1.79, 0.606, 0.848.*

[8]

20.) (a) $\det \boldsymbol{A} = 5(1) - 7(-2) = 19$

$\boldsymbol{A}^{-1} = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$ (A2)

Note: *Award (A1) for* $\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix}$, *(A1) for dividing by 19.*

OR

$\boldsymbol{A}^{-1} = \begin{pmatrix} 0.0526 & 0.105 \\ -0.368 & 0.263 \end{pmatrix}$ (G2) 2

(b) (i) $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C} \Rightarrow \boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B}$ (M1)

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ (A1)

OR

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ (A2)

(ii) $(\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} = \begin{pmatrix} -11 & -7 \\ -13 & 9 \end{pmatrix}\begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$ (A1)

$\Rightarrow \boldsymbol{X} = \begin{pmatrix} \frac{38}{19} & \frac{-57}{19} \\ \frac{-76}{19} & \frac{19}{19} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$ (A1)

OR

$\boldsymbol{X} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$ (G2) 4

Note: *If premultiplication by* $\boldsymbol{A}^{-1}$ *is used, award (M1)(M0) in part (i) but award (A2) for* $\begin{pmatrix} \frac{-37}{19} & \frac{11}{19} \\ \frac{12}{19} & \frac{94}{19} \end{pmatrix}$ *in part (ii).*

[6]

21.) (a) $M^2 = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} a^2+4 & 2a-2 \\ 2a-2 & 5 \end{pmatrix}$ (A1)(A1)(A1)(A1) 4

(b) $2a - 2 = -4$

$\Rightarrow a = -1$ (A1)

Substituting: $a^2 + 4 = (-1)^2 + 4 = 5$ (A1) 2

Note: *Candidates may solve* $a^2 + 4 = 5$ *to give* $a = \pm 1$*, and then show that only* $a = -1$ *satisfies* $2a - 2 = -4$*.*

(c) $M = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$

$M^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$ (M1)

$= \frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$ (A1)

$-x + 2y = -3$

$2x - y = 3$

$\Rightarrow \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ (M1)(M1)

$\Rightarrow \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ (A1)

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ (A1) 6

ie $x = 1$

$y = -1$

Note: *The solution must use matrices. Award no marks for solutions using other methods.*

[12]